Interface between Global Optimization Problems and Computer Science

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Abstract:**The subject of this paper is a relation between two disciplines and to give a basic idea about global optimization and its solution problems. Operations research emerged out of optimization questions in warfare logistics in the 1940's and quickly established itself as one of the cornerstones of industrial mathematics.We cannot possibly give a complete overview of all interfaces between operations research and computer science.The time is too short and our background is too limited so please expect a biased view of some interfaces.**

KeyWords: **Global optimization: Constrained, unconstrained problems, Genetic Algorithm.**

I. Introduction

$x \in C(I, R^n)$).When x is in a space of functions, it is necessary to specify what kind of functions, for example $x \in V = C(I, R^n)$.

We now present some definitions concerning properties of the values that f(x), $x \in I$ can assume.

Definition 1. f has a global minimum at $x_1 \in I$ if

$$f(x_1) \leq f(x) \text{ for all } x \in I.$$

(A global minimum is sometimes called an absolute minimum.)

Definition 2.f has a global maximum at $x_1 \in I$ if

$$f(x_1) \geq f(x) \text{ for all } x \in I.$$

(A global maximum is sometimes called an absolute maximum.)

Definition 3. f has a global extremum at $x_1 \in I$ if f either a global minimum or a global maximum at x_1.(A global extremum is sometimes called an absolute extremum.)

Definition 4.f has a local minimum at $x_1 \in I$ if there exists a $\delta \in R^+$ such that

$$f(x_1) \leq f(x) \text{ for all } x \in I \text{ satisfying } |x - x_1| < \delta$$

(A local minimum is sometimes called a relative minimum).

Definition 5.f has a local maximum at $x_1 \in I$ if there exists a $\delta \in R^+$ such that

$f(x_1) \geq f(x)$ for all $x \in I$ In each optimization problem considered, it is required to find an optimum (minimum or maximum) of a function f(x), where x is a vector, or function, in some specified class V, and is subject to constraints- equations or inequalities –which may be symbolized by $x \in K$,where K is the feasible set (or constraint set). In particular, x may be a vector of n components (thus, $x \in R^n$ the Euclidean space of n dimensions), or x may be a continuous function from an interval I=[a,b] into R^n (denoted by satisfying

$$|x - x_1| < \delta$$

(A local maximum is sometimes called a relative maximum).

Definition 6. f has a local extremum at $x_1 \in I$, if *f* has either a local minimum or a local maximum at x_1.(A global extremum is sometimes called an relative extremum).

Definition 7.f has a stationary point at $x_1 \in I$ if f is differentiable at x_1 and

$$f'(x_1) = 0$$

(A stationary point is called a critical point by some authors.)

The interface between computer science and operations research has drawn much attention recently especially in optimization which is a main tool in operations research. In optimization area, the interest on this interface has rapidly increased in the last few years in order to develop nonstandard algorithms that can deal with optimization problems which the standard optimization techniques often fail to deal with. Global optimization problems represent a main category of such problems. Global optimization refers to finding the extreme value of a given non-convex function in a certain feasible region and such problems are classified in two classes; unconstrained and constrained problems. Solving global optimization problems has made great gain from the interest in the interface between computer science and operations research.

In general, the classical optimization techniques have difficulties in dealing with global optimization problems. One of the main reasons of their failure is that they can easily be entrapped in local minima. Moreover, these techniques cannot generate or even use the global information needed to find the global minimum for a function with multiple local minima.

The interaction between computer science and optimization has yielded new practical solvers for global optimization problems, called metaheuristics. The structures of metaheuristics are mainly based on simulating nature and artificial intelligence tools. Metaheuristics mainly invoke exploration and exploitation search procedures in order to diversify the search all over the search space and intensify the search in some promising areas. Therefore, metaheuristics cannot easily be entrapped in local minima. However, metaheuristics are computationally costly due to their slow convergence. One of the main reasons for their slow convergence is that they may fail to detect promising search directions especially in the vicinity of local minima due to their random constructions.

Many recent problems in science, engineering and economics can be expressed as computing globally optimal solutions [16, 17, 20, 21, 22]. Using classical nonlinear programming techniques may fail to solve such problems because these problems usually contain multiple local optima. Therefore, global search methods should be invoked in order to deal with such problems. In recent years, there has been a great deal of interest in emerging some artificial intelligence tools in the area of optimization. These tools which are normally calledmet heuristics are mainly proposed by simulating nature or by invoking intelligent learned procedures [6,19, 23].

One main category of global optimization problems contains the problems which are characterized by one or more of the following properties:

• Calculation of the objective function (or constraint functions if exist) is very expensive or time consuming.

• The exact gradient of the objective function (or constraint functions if exist) cannotbe computed, or its numerical approximation is very expensive or time consuming.

• The values of the objective function (or constraint functions if exist) contain noise.

Such problems exist in many real-world applications and achieving the exact global solution is neither possible nor desirable. Therefore, using derivative-free global search methodsis highly needed in order to achieve acceptable solutions. Actually, metaheuristicsfightcourageously when applied to these problems and they could obtain highly accurate solutionsin many cases [19]. The power of met heuristics comes from the fact that they are robustand can deal successfully with a wide range of problem areas. However, these methods,especially when they are applied to complex problems, suffer from the slow convergence thatbrings about the high computational cost. The main reason for this slow convergence is thatthese methods explore the global search space by creating random movements without usingmuch local information about promising search direction. In contrast, local search methodshave faster convergence due to their using local information to determine the most promisingsearch direction by creating logical movements. However, local search methods can easilybe entrapped in local minima.

One approach that recently has drawn much attention is to combine metaheuristicswithlocal search methods to design more efficient methods with relatively faster convergence thanthe pure met heuristics, see [8, 9, 10, 11, 12, 13] and references therein. Moreover, these hybrid methods are not easily entrapped in local minima because they still maintain the merits of the metaheuristics. New hybrid methods that combine metaheuristics and direct search methods can be proposed in order to deal with the global optimization problems that have the abovecharacteristics. Specifically, local search guidance in the direct search methods is invoked todirect and control the global search features of metaheuristics to design more efficient hybridmethods.

Unconstrained Problem

$$\min_{x \in R^n} f(x)$$

where f is a generally nonconvex, real valued function defined on R^n.

Constrained Problem

$$\min_{x} f(x),$$

Such that $g_i(x) \le 0, i = 1,......., l$,

$h_j(x) = 0, j = 1......, m, x \in S$,

where f, g_i and h_j are real-valued functions defined on the search space $S \subseteq R^n$. Usually, the search space S is defined as $\{x \in R^n : x_i \in [l_i, u_i], i = 1....., n\}$.

Met heuristics

The term "metaheuristics" was first proposed by Glover [5]. The word "metaheuristics"contains all heuristics methods that show evidence of achieving good quality solutions forthe problem of interest within an acceptable time. Usually, metaheuristics offer no guaranteeof obtaining the global solutions.

Metaheuristics can be classified into two classes; population-based methods and point to-point methods. In the latter methods, the search invokes only one solution at the endof each iteration from which the search will start in the next iteration. On the other hand,the population-based methods invoke a set of many solutions at the end of each iteration.

Below, we highlight the principles of genetic algorithm as an example of population-based methods, and simulated annealing and tabu search as examples of point-to-point methods.

Genetic Algorithms

A genetic algorithm (GA) is a procedure that tries to mimic the genetic evolution of a species.Specifically, GA simulates the biological processes that allow the consecutive generations in apopulation to adapt to their environment. The adaptation process is mainly applied through genetic inheritance from parents to children and through survival of the fittest. Therefore,GA is a population-based search methodology. Some pioneering works traced back to the middle of 1960s preceded the main presentation of the GAs of Holland [15] in 1975. However,GAs were limitedly applied until their multipurpose presentation of Goldberg [7] in search,optimization, design and machine learning areas. Nowadays, GAs are considered to be themost widely known and applicable type of met heuristics [2, 3, 18].GA starts with an initial population whose elements are called chromosomes. The chromosome consists of a fixed number of variables which are called genes. In order to evaluatesand rank chromosomes in a population, a fitness function based on the objective functionshould be defined. Three operators must be specified to construct the complete structureof the GA procedure; selection, crossover and mutation operators. The selection operatorcares with selecting an intermediate population from the current one in order to be used bythe other operators; crossover and mutation. In this selection process, chromosomes withhigher fitness function values have a greater chance to be chosen than those with lower fitnessfunction values. Pairs of parents in the intermediate population of the current generationare probabilistically chosen to be mated in order to reproduce the offspring. In order to increase the variability structure, the mutation operator is applied to alter one or more genesof a probabilistically chosen chromosome. Finally, another type of selection mechanism isapplied to copy the survival members from the current generation to the next one.

GA operators; selection, crossover and mutation have been extensively studied. Manyeffective setting of these operators have been proposed to fit a wide variety of problems

Fitness Function

Fitness function F is a designed function that measures the goodness of a solution. It shouldbe designed in the way that better solutions will have a higher fitness function value thanworse solutions. The fitness function plays a major role in the selection process.

Coding

Coding in GA is the form in which chromosomes and genes are expressed. There are mainlytwo types of coding; binary and real. The binary coding was presented in the GA original presentation [15] in which the chromosome is expressed as a binary string. Therefore, the search space of the considered problem is mapped into a space of binary strings through acoder mapping. Then, after reproducing an offspring, a decoder mapping is applied to bringthem back to their real form in order to compute their fitness function values. Actually,many researchers still believe that the binary coding is the ideal. However, the real codingis more applicable and easy in programming. Moreover, it seems that the real coding fits the continuous optimization problems better than the binary coding [14].

Selection

Consider a population P, selection operator selects a set $P' \subseteq P$ of the chromosomes thatwill be given the chance to be mated and mutated. The size of P′ is the same as that ofP but more fit chromosomes in P are chosen with higher probability to be included in P′.

Therefore, the most fit chromosomes in P may be represented by more than one copy in P′and the least fit chromosomes in P may be not represented at all in P′.

The most common selection operators are the proportional selection [15] and linear ranking selection [4], see [1] for ease of explanation and comparison of different selection operators.

Crossover and Mutation

Crossover operator aims to interchange the information and genes between chromosomes.Therefore, crossover operator combines two or more parents to reproduce new children,then, one of these children may hopefully collect all good features that exist in his parents.

Crossover operator is not typically applied for all parents but it is applied with probabilitypc which is normally set equal to 0.6. Actually, crossover operator plays a major role inGA, so defining a proper crossover operator is highly needed in order to achieve a better performance of GA. Different types of crossover operators have been studied, see [14] as acondensed survey.

Mutation operator alters one or more gene in a chromosome. Mutation operator aimsto achieve some stochastic variability of GA in order to get a quicker convergence. Theprobability pm of applying the mutation operator is usually set to be small, normally 0.01.

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